

UNCLASSIFIED

Defense Technical Information Center  
Compilation Part Notice

ADP012729

TITLE: On the Applicability of Miniband Transport in Semiconductor Superlattices

DISTRIBUTION: Approved for public release, distribution unlimited  
Availability: Hard copy only.

This paper is part of the following report:

TITLE: Nanostructures: Physics and Technology International Symposium [6th] held in St. Petersburg, Russia on June 22-26, 1998 Proceedings

To order the complete compilation report, use: ADA406591

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:

ADP012712 thru ADP012852

UNCLASSIFIED

## On the applicability of miniband transport in semiconductor superlattices

*Andreas Wacker*<sup>†</sup> and *Antti-Pekka Jauho*<sup>‡</sup>

<sup>†</sup> Institut für Theoretische Physik, Technische Universität Berlin

<sup>‡</sup> Mikroelektronik Centret, Danmarks Tekniske Universitet

**Abstract.** Electrical transport in semiconductor superlattices is often described by a miniband model, where the electrons perform a quasiclassical motion. Although this model is assumed to be valid in the limit of a large miniband width, some experiments indicate its validity for narrow minibands as well. By a comparison with a full calculation based on nonequilibrium Green functions we show, that this is the case if the width of the electronic distribution is larger than both the field drop per period and the scattering width. In addition we show that simplified expressions for miniband transport and sequential tunneling become identical in this regime.

The fabrication of semiconductor superlattices allows for the study of electric transport in an artificial band structure [1, 2, 3]. Spectacular effects like negative differential conductivity [4] and Bloch oscillations [5] have been observed experimentally. Recently, high frequency oscillations due to travelling field domains have been studied [6, 7] as well, see also Ref. [8] and references cited therein. For low electric fields (where higher subbands are not involved) the transport in superlattices is usually described by the semiclassical motion of electrons in the miniband [1]. This approach is assumed to be valid if the miniband width  $\Delta$  is significantly larger than the scattering width  $\Gamma$  [3]. Here we want to investigate its applicability by a direct comparison with a full quantum transport model.

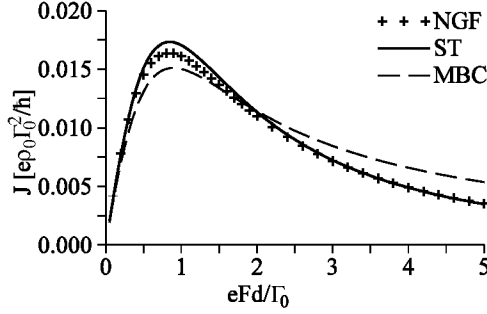
We have recently proposed a transport model based on nonequilibrium Green functions in order to study transport in superlattices [9]. The current density is determined

$$J(F) = \frac{e}{2\pi^2} \int d^2k \frac{2}{\hbar} \text{Re} \left\{ \frac{\Delta}{4} G_{n+1,n}^<(t, t, \mathbf{k}) \right\}. \quad (1)$$

First we calculate the retarded and advanced Green functions  $G_{m,n}^{\text{ret}}$  and  $G_{m,n}^{\text{adv}}$  connecting different wells  $n, m$ . We use self energies  $\Sigma^{\text{ret}}$  and  $\Sigma^{\text{adv}}$  which include both the coupling between the wells (which is equal to  $\Delta/4$ ) and scattering at  $\delta$ -potentials within the self-consistent Born approximation. Then we determine the lesser Green function between neighbored wells via the Keldysh equation

$$G_{m,n}^<(\mathcal{E}, \mathbf{k}) = \sum_{m_1} G_{m,m_1}^{\text{ret}} \Sigma_{m_1}^< G_{m_1,n}^{\text{adv}} \quad (2)$$

where  $\Sigma^<$  is obtained from a kind of relaxation time approximation. This approach allows the treatment of coupling and scattering on equal footing and therefore goes beyond the restrictions of standard transport models like miniband transport or sequential tunneling. Details of the calculation are given in Ref. [9]. The resulting current-field



**Fig 1.** Current-field relation for three different approaches: sequential tunneling (full line), miniband conduction (dashed line), and our quantum transport theory (crosses). Parameters:  $\Delta = 2\Gamma$ ,  $k_B T = 3\Gamma$ ,  $N_{2D} = 0.2\rho_0\Gamma$ .

relations are found to be in good agreement with the results from the miniband conduction model if both  $\Delta/2 \gg \Gamma$  and  $\Delta/2 \gg eFd$  are fulfilled [9]. (Here  $eFd$  denotes the potential drop per period.) Otherwise miniband conduction fails for a low electron concentration and a low temperature. In the opposite limit of small minibands ( $\Delta/2 \ll \Gamma$ ) the results of our quantum transport model reproduce the results obtained from the sequential tunneling model described in Refs. [8].

Experiments [10, 11] indicate that miniband conduction holds for superlattices with relatively narrow miniband widths of  $\Delta \approx 3\text{--}4$  meV as well, which are of the order typical scattering widths  $\Gamma \sim 2\text{--}3$  meV [11]. These experiments have been performed at temperatures above 70 K, i.e.  $k_B T > 6$  meV, which is larger than both  $\Delta/2$  and  $\Gamma$ . Therefore we have performed several calculations within our full quantum transport theory for the experimental situation  $k_B T \gg \Delta/2, eFd, \Gamma$  and compared the current field relations with the respective ones obtained from the miniband conduction model and sequential tunneling. As can be seen in Fig. 1 all three approaches give almost identical results in this regime.

In order to get more insight into these findings we want to consider the models for sequential tunneling and miniband conduction in detail. The miniband model is usually evaluated assuming a constant scattering time  $\tau$  within the relaxation time approximation. In the nondegenerate limit one obtains the current density [3]

$$J_{\text{mini}}(F) = eN_{2D} \frac{I_1(\Delta/2k_B T)}{I_0(\Delta/2k_B T)} \frac{\Delta\tau}{2\hbar^2} \frac{eFd}{(eFd\tau/\hbar)^2 + 1} \quad (3)$$

where  $N_{2D}$  is the electron density per period of the superlattice and  $I_0, I_1$  are the modified Bessel functions. For  $k_B T \gg \Delta/2$  the ratio between the Bessel functions becomes  $\Delta/4k_B T$  and the current can be written as

$$J_{\text{mini}}(F) = eN_{2D} \frac{\Delta^2\tau}{8k_B T\hbar^2} \frac{eFd}{(eFd\tau/\hbar)^2 + 1} \quad \text{for } k_B T \gg \Delta/2. \quad (4)$$

Now let us regard sequential tunneling between the wells. As the coupling between neighbored quantum wells is given by  $\Delta/4$  the transition rate scales with  $\Delta^2$ . The current field relation is determined by a competition between the resonance condition and

the difference of chemical potentials between neighboring wells. Assuming a constant broadening  $\Gamma$  of the states the current can be approximated by [8]

$$J_{\text{seq}}(F) = e\rho_0 \frac{\Delta^2}{8\hbar} \frac{\Gamma}{(eFd)^2 + \Gamma^2} \int_0^\infty dE [n_F(E - \mu) - n_F(E + eFd - \mu)] \quad (5)$$

where  $n_F(x) = [\exp(x/k_B T) + 1]^{-1}$  is the Fermi function,  $\mu$  is the chemical potential with respect to the bottom of the well and  $\rho_0 = m/(\pi\hbar^2)$  is the two-dimensional density of states. In the nondegenerate limit we have

$$\begin{aligned} & [n_F(E - \mu) - n_F(E + eFd - \mu)] = \\ & \frac{N_{2D}}{\rho_0 k_B T} \left[ \exp\left(-\frac{E}{k_B T}\right) - \exp\left(-\frac{E + eFd}{k_B T}\right) \right] \end{aligned} \quad (6)$$

and the integration yields

$$J_{\text{seq}}(F) = eN_{2D} \frac{\Delta^2}{8k_B T \hbar} \frac{\Gamma eFd}{(eFd)^2 + \Gamma^2} \quad \text{for } k_B T \gg eFd. \quad (7)$$

Comparison between Eq. (4) and Eq. (7) exhibits that the expressions are identical if we set  $\Gamma = \hbar/\tau$ . Note that the origin of  $\tau$  and  $\Gamma$  is completely different:  $\tau$  describes the relaxation time of electrons in the miniband while  $\Gamma$  is the energy width of localized states in the wells. Nevertheless, both result from scattering processes and one can explicitly show the equality  $\Gamma = \hbar/\tau$  for scattering at  $\delta$ -potentials [9]. From these observation we have found, that in the nondegenerate limit the simplified models for sequential tunneling and miniband conduction yield exactly the same relations if  $k_B T \gg \Delta/2, eFd$ . In particular the current density scales with the square of the miniband width and is inversely proportional to the temperature as observed in Ref. [11] and Ref. [10], respectively. For a nondegenerate electron gas, one can similarly show that  $J_{\text{mini}} = J_{\text{seq}}$  for  $\mu \gg \Delta/2, eFd$  (see Refs. [8]). Again comparison with the full quantum model indicates that both approaches are valid for  $\mu \gg \Delta/2, eFd, \Gamma$ .

Note that all the approaches considered here treat scattering only in a phenomenological way and do not properly take into account energy relaxation processes. Therefore a full quantitative agreement with the experimental data can not be achieved. Nevertheless, we are convinced that the essential interplay between scattering and coupling is treated correctly and that we can draw conclusions concerning the applicability of the models. In particular we find that both miniband conduction and sequential tunneling are appropriate to describe the electric transport in superlattices if the energy width of the carrier distribution,  $\text{Max}(\mu, k_B T)$ , is large compared to the scattering width, the potential drop per period, and the miniband width.

## References

- [1] L. Esaki and R. Tsu, *IBM J. Res. Develop.* **14** 61 (1970).
- [2] R. F. Kazarinov and R. A. Suris, *Sov. Phys. Semicond.* **6** 120 (1972), [*Fiz. Tekh. Poluprov.* **6** 148 (1972)].
- [3] A. Y. Shik, *Sov. Phys. Semicond.* **8** 1195 (1975), [*Fiz. Tekh. Poluprov.* **8** 1841 (1974)].
- [4] A. Sibille, J. F. Palmier, H. Wang, and F. Mollot, *Phys. Rev. Lett.* **64** 52 (1990).
- [5] C. Waschke, H. G. Roskos, K. Schwedler, K. Leo, H. Kurz, and K. Köhler, *Phys. Rev. Lett.* **70** 3319 (1993).

- [6] J. Kastrup, R. Hey, K. H. Ploog, H. T. Grahn, L. L. Bonilla, M. Kindelan, M. Moscoso, A. Wacker, and J. Galán, *Phys. Rev. B* **55** 2476 (1997).
- [7] E. Schombug, K. Hofbeck, J. Grenzer, T. Blomeier, A. A. Ignatov, K. F. Renk, D. G. Pavelev, Y. Koschurinov, V. Ustinov, A. Zhukov, S. Ivanov, and P. S. Kopev, *Appl. Phys. Lett.* **71** 401 (1997).
- [8] A. Wacker, in *Theory of transport properties of semiconductor nanostructures*, edited by E. Schöll (Chapman and Hall, London, 1998).
- [9] A. Wacker and A.-P. Jauho, *Phys. Rev. Lett.* **80** 369 (1998).
- [10] G. Brozak, M. Helm, F. DeRosa, C. H. Perry, M. Koza, R. Bhat, and S. J. Allen, *Phys. Rev. Lett.* **64** 3163 (1990).
- [11] A. Sibille, J. F. Palmier, M. Hadjazi, H. Wang, G. Etemadi, E. Dutisseuil, and F. Mollot, *Superlattices and Microstructures* **13** 247 (1993).